Problem solving strategies in math anxiety management

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Abstract: Math anxiety is a phenomenon that overshadows the lives of many individuals, evoking strong emotions in students, teachers and parents blocking academic progress, and leading to reduced career opportunity choices. Anxiety associated with mathematics is a persistent challenge to the teaching profession across many sectors and academic levels. The rationale for this research is to better understand math anxiety by identifying key risk factors for math anxiety and appropriate strategies to reduce it. Results confirmed the highly complex multidimensional nature of math anxiety and the power that a single strategy could address multiple risk factors. Mathematical content knowledge of teachers is a highly significant component of sector-wide strategies to deal with math anxiety. Teacher professional development, increased awareness of the power of informal methods of mathematical problem-solving and multiple approaches to mathematical problem solving is needed to support efforts to reduce math anxiety.

Keywords: Math anxiety; Mathematics; Risk Factors; Problem-solving strategies

Introduction

When students study mathematics at school or in a tertiary education environment they face the challenge of learning a new language, the language of mathematics. It is commonplace for children and adults to feel anxious about acquiring mathematical knowledge and developing mathematical problems-solving skills. Since mathematics has such a wide application in life situations, succeeding or failing can have long-term consequences, such as lower self-confidence and reduced career opportunities. This review of literature aims to propose a model to describe the relationship between risk factors and problem-solving strategies in mathematical contexts.

Math anxiety definitions

Several definitions of math anxiety have been developed which highlight the range and intensity of negative emotions involved. Simply, it can be conceived as performance anxiety associated with mathematical cognition and activities. Ashcraft (2002) defines it as “a feeling of tension, apprehension, or fear that interferes with math performance” (p. 181). Such feelings have a negative effect upon the sufferer’s physiological, affective and cognitive states disrupting thinking processes associated with problem-solving. Biggs and Preis (2001) (as cited in Benner, 2010) stress “the panic, helplessness and anxiety that some individuals experience when required to solve a mathematical problem” (p. 6). The belief that it could be a learned behaviour having its origins in negative early learning experiences of mathematics, is well-supported by the literature (Hembree 1990; Rossman 2006; Gresham 2009 & Klinger, 2009). McAnallen (2010) draws attention to other types of math anxiety such as mathematics test anxiety, numerical anxiety (anxiety associated with the manipulation of numbers) and abstraction anxiety. Onwuegbuzie (2004) also identifies statistics anxiety. The experience of anxiety in a broad range of mathematics settings, both formal and informal (Richardson & Suinn, 1972), and across a variety of topics underlines the pervasiveness of this issue.
Math anxiety risk factors

There are many predisposing factors for math anxiety. In a study to understand the causes of negative beliefs about mathematics in pre-service teachers, Uusimaki and Nason (2004) highlight the complex and intricate nature of math anxiety. The factors that may increase risk include the academic level of the mathematics, the topic, being observed during mathematical activities, the form of assessment, the percentage weighting of the assessment, whether the assessment is formative or summative, the teacher’s attitude to the subject and the personality traits of the student. The level of the teacher’s math anxiety may also contribute to the student’s anxiety levels.

Intrinsic risk factors

There appears to be a contradiction between adults who resist learning at school but exercise surprising competence in mathematical tasks during everyday situations Wedege and Evans (2006). In university students studying commerce subjects, Yenilmez, Girginer and Uzun (2007) found that attitude affected how frequently students engaged in mathematics, the level of enjoyment associated with it and how well they did. Shen (2009) identifies that attitudes about mathematics are correlated with cognitive and affective elements of math anxiety and that these are learned behaviours (Furlich & Dwyer, 2007) interrelating in complex ways. Many students believe that mathematics problem have one correct answer, that working hard cannot compensate for lack of innate ability, and that mathematics is mainly a collection of facts and strict procedures that must be memorised (McKinney, 2005). Deep and intense emotions are common in the study of mathematics. Despite common beliefs about the purely rational nature of mathematics, adults frequently experience emotionally intense reactions to mathematics (Bibby, 2002). Criticism of their mathematical knowledge and skills triggers the expression of shame. The enduring myth of mathematics being a cold, unemotional and abstract discipline of the mind persists (Usop, Sam, Sabri & Wah, 2009) and contributes to math anxiety.

The perceived difficulty of mathematics varies between individuals and can be related to topic areas. There is evidence that some topics are commonly believed to be very difficult and tend to evoke high math anxiety. The transition from primary to secondary schooling is accompanied by the introduction of topics like algebra and trigonometry. Lane (2011) found that algebra frequently evokes fear, and students tend to associate it with meaningless abstraction using letters in the place of numbers, causing increased confusion and stress. Jackson and Ginsburg (2008) identify the mystique associated with algebra as an overwhelming abstraction.

Extrinsic risk factors

The teacher’s influence upon the students’ anxiety levels in mathematics is fairly well documented. Rule and Harrell (2006) found a link between students’ math anxiety and the teacher’s anxiety levels. This may feedback into the teaching as a self-reinforcing cycle of increasing math anxiety for student and teacher.

Metje, Frank, and Croft (2007) identify the key role that the lecturer’s motivation of the students plays as part of negative and positive cycles of learning in mathematics. A tension exists between setting high expectations to challenge learners on one hand, and managing the stress levels in the learning environment on the other. Ginsburg (2008) identifies the attempt to dissipate math anxiety at all costs to be a risky instructional practice. To remove the
struggle for a solution to a problem or to protect learners from being wrong can reinforce the sense of helplessness that math anxiety is associated with. Some teachers are also affected by math anxiety and they have a central role to play in the complex challenge of managing math anxiety for themselves and for their students. One key challenge facing teachers, especially at the primary and intermediate levels, is their own weak mathematical content knowledge. This is an important risk factor for the teacher’s math anxiety.

Mathematical content knowledge varies widely among teacher trainees, especially at primary and intermediate levels. One example is a negative relationship between teachers’ math anxiety and their conceptual knowledge of fractions (Rayner, Pitsolantis & Osana 2009). Teachers with a poor knowledge of mathematics experience difficulty in offering full and deep explanations of content to students. Highly knowledgeable and confident teachers, may have difficulty appreciating that mathematics is a hard subject for many Newell (2011).

Mathematics content knowledge itself is not sufficient, in itself, for successful teaching (Silverman & Thompson, 2008). The ability to graduate in mathematics qualifications is no guarantee of a strong ability to teach mathematics effectively.

Strategies for dealing with Math anxiety

Some strategies for dealing with math anxiety, especially as they relate to mathematical problem solving will now be set forth. First, a basis for categorising such strategies into intrinsic and extrinsic will be outlined, and second, a selection of extrinsic and intrinsic strategies will be outlined.

The blend of problem solving strategies to help ease math anxiety will vary from person to person. Knollman and Wild (2007) studying the effects of parental support upon student’s emotions during homework, distinguish between intrinsic and extrinsic motivations. Amelink (2012), in particular, identifies intrinsic and extrinsic factors that influence female interest in mathematics, linking these to strategies and interventions to address math anxiety. Viewing strategies for dealing with math anxiety from personal and environmental perspectives simplifies analysis and supports shared responsibility for its management.

Extrinsic strategies

A blend of teaching methods, classroom practice, one-on-one tutoring of needy students with reflections on the history and importance of mathematics, contributed to reductions in math anxiety among math anxious pre-service elementary teachers (Ashon and Reinink, 2009). This holistic and supportive style improved their confidence in the mathematics classroom.

Engaging students in mathematics problem solving sometimes requires using alternative strategies involving more kinaesthetic approaches to learning. Mathematics typically involves visualisation, conceptualization, and abstraction. Mickleson and Ju (2011) found that using visualisation and performing geometric transformations through artistic dance and movement, made the learning experience more enjoyable, less abstract and more engaging than traditional approaches. Gadandidis and Borba (2008) believe that if teachers can call students ‘young mathematicians’ and help them view mathematics as another of the arts, it could benefit alienated learners. Mathematics can then be seen as human activity that all can contribute to, and be actively involved in during their everyday lives. Some topics also may not naturally lend themselves to dramatic representation, and text books can potentially overwhelm.
students textually. For other students conventional direct instruction for short periods in a way that relates to everyday life and language can be more productive.

Making connections between informal and formal viewpoints in mathematics by use of analogy and valuing informal problem solving strategies, are two useful approaches for reducing math anxiety. Fuson, Kalschman & Bransford (2005) emphasise the importance of allowing students to use their own informal problem solving strategies, facilitating talking about mathematics, and instructional activities which aim to bridge students’ informal knowledge with formal mathematics concepts. Common sense is also a very important element of adult problem solving, especially involving mathematics. Colleran and O’Donoghue (2007) exploring the relationship between quantitative problem solving and common sense found that “common sense is a powerful intellectual resource and provides the bedrock on which mathematical understanding is built.” (p. 20). Clearly, mathematics teachers must explicitly value students’ common sense to help enhance problem solving confidence and reduce levels of math anxiety.

Students frequently employ multiple problem solving strategies when faced with mathematical tasks. They also differ in how much of the processing is performed mentally, in written working, through visual representations, doodles and through talk with their peers. Thus, in addition to imparting a range of standard problem solving techniques, teachers must learn to trust students to work with approaches that make sense to them and express their intuitive understandings of the problems (Johanning, 2004). Star and Rittle-Johnson (2007) found that exposing students to multiple strategies develops increased flexibility and control in solving mathematical problems.

Teaching styles which support reflective classroom discussion and deal creatively with mathematical errors have been associated with better student understanding and attitudes to mathematics (Swan, 2006). Melis (2004) identified several benefits when teachers engage students with mathematical errors, viewing them as opportunities to stimulate exploration, develop an awareness of multiple solutions, increase verbalising of mathematical cognition and sensitivity to careful reasoning.

Several extrinsic strategies related to mathematical problem solving have been considered to alleviate math anxiety. The teacher’s content knowledge is a highly significant risk factor. Other strategies that were considered included kinaesthetic approaches to learning, making meaningful connections between informal and formal aspects of mathematics, valuing common sense, employing multiple (often combined) approaches to problem solving, use of video multimedia to illustrate concepts, and using a systematic approach to solving problems. In the next section we will consider intrinsic strategies for managing math anxiety.

**Intrinsic strategies**

The distinction between extrinsic and intrinsic is not always obvious: the teacher-student dynamic is much more subtle and complex than a simple two category analysis. In light of the literature surveyed so far, there is evidence for considerable overlap between extrinsic and intrinsic strategies. However, by referring to strategies as intrinsic we emphasise that students have responsibility to control their part of the student-teacher relationship.

Students engage in mathematical problem solving using a range of strategies related to their learning styles. Visual representations can be very helpful to students and teachers in working together to make sense of problems and support conceptual thinking. Elia and Phillipou (2004) found that pictures serve certain useful functions in problem solving including:
“representational, organizational and informational” (p. 327). When students generate their own representations of problems by a sketch, diagram or doodle they may feel freer to explore the affective side of mathematical cognition and express the problem in more informal language. Walker (2007) found that effective problem solvers are able to use diagrams and pictures to illustrate the relationships between the various parts of a problem. However, some mathematical problems may have more subtle features, or levels of abstraction that may not be able to be represented in picture form precisely enough, requiring the use of other strategies. When a number of strategies have been exhausted the student needs to be able to persevere to solve the problem.

Johnston-Wilder and Lee (2010) identify mathematical resilience whereby students develop persistence in their efforts in problem solving, deal with their emotions during failure and maintain a creative and positive disposition. Shum and Crick (2012) also highlight resilience, as a crucial component in which students draw on a broad repertoire of mathematical problem solving strategies to manage their math anxiety.

A key skill is the ability to generate or apply informal strategies effectively. Koedinger and Corbett (2006) identify working backwards from the desired result, trial and error, and the use of stories to make sense of simple algebra problems. Interestingly, they found that more students experienced difficulty solving mathematical problems in conventional symbolic form, than when set within a story context. These informal often hidden strategies work well-enough for students and provide a more gradual bridge to formal algebra.

Visual representation, mathematical resilience, seeking help and the use of informal problem-solving strategies have the potential to assist learners to address their math anxiety. Extrinsic and intrinsic strategies outlined above offer two key perspectives on the challenge of math anxiety. In the next section the relationship between strategies and risk factors will be discussed.

Discussion

This research highlights the subtle and complex interplay between a range of math anxiety risk factors and strategies used to address them. Identifying formal and informal contexts in which math anxiety occurs emphasises the pervasive nature of this phenomenon. Throughout the literature surveyed, maintaining a clear separation between intrinsic and extrinsic perspectives has been problematic thus far.

Risk factors continuum

Risk factors in Figure I can range from extrinsic to intrinsic. External factors such as the classroom environment is viewed as more extrinsic than intrapersonal components of the individual which are viewed as intrinsic.

Strategies continuum

Strategies in Figure I can likewise range from extrinsic to intrinsic. Instruction in systematic approaches to problem solving and the teacher making explicit connections between the informal and formal mathematical knowledge of students could be viewed as more at the extrinsic end of the continuum. Collaborative strategies where teachers and students work together in small groups to solve mathematical problems may be positioned more toward the more intrinsic end of the continuum.
Figure 1: Math anxiety Risk factors and Strategies Continua
Risks and strategies continua

A simple model was developed to illustrate the relationship between math anxiety risk factors and problem-solving strategies depicted in Figure 1. This continua suggests the existence of subtle combinations of risk factors and strategies by placing them in one of the quadrants. A more detailed mapping of specific risk factor/strategy combinations was developed as shown in Table I and Table II below. Natural links among risk factors and among strategies are evident. For example, the extrinsic risk factors of teaching style and teacher’s classroom culture are naturally associated. A positive, encouraging teaching style which accepts mistakes as perfectly natural occurrences from which students and teachers can learn, form part of effective classroom cultures. Amongst intrinsic risk factors students’ emotions and self-confidence / perception can be considered together. For example, self-regulation of student emotion could be associated with increased self-confidence in study.

The literature consulted in this review supports the idea that a single problem solving strategy, whether extrinsic or intrinsic, has the potential to address multiple risk factors. To illustrate this we will take a selection of key strategies from each table, and consider their potential in dealing with math anxiety risk factors. We will first consider the teacher’s subject knowledge, since poor content knowledge can be thought of as an intrinsic risk factor for the teacher’s math anxiety and an extrinsic risk factor for the student’s math anxiety. It is also a key issue that needs addressing in the teaching profession’s efforts to deal with math anxiety.

Table I: Potential Extrinsic Math anxiety Risk factors and Strategies

<table>
<thead>
<tr>
<th>Risk factors</th>
<th>Teacher’s Motivation to teach mathematics</th>
<th>Teacher’s Classroom Culture</th>
<th>Teaching Style</th>
<th>Teacher’s Mathematical Content Knowledge</th>
<th>Student’s Mathematical Symbols Anxiety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enthusiasm for subject</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Kinesthetic Approaches</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Connecting Informal &amp; Formal</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Valuing Common sense</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Multiple strategies</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Computer Technology</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Systematic Problem Solving</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Teaching Styles</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Support Discussion</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Cooperative Classroom Culture</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>De-Mystification of Symbols</td>
<td>_</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
Teacher’s subject knowledge

The results from the investigation involving Table I indicate that poor mathematical content knowledge can be associated with negative attitudes to the subject and increased teacher Math anxiety. With appropriate training to improve mathematical content knowledge, teachers were more confident, had less math anxiety and began to develop more positive attitudes to mathematics. There is potential for teachers to develop more motivation to teach mathematics, foster a positive and collaborative classroom culture, eliminate weaknesses in their teaching style and be able to confidently assist students to overcome their anxiety about mathematical symbols. Thus, one extrinsic strategy can address many extrinsic risk factors.

Connecting informal and formal

Making meaningful contextual connections between informal, everyday mathematics and formal mathematics content stands out as a key strategy in addressing math anxiety. Teachers who use this strategy to clarify mathematical concepts find this supports the development of their content knowledge, potentially leading to increased motivation to teach mathematics and share the benefits of their insights with students.

Multiple problem solving strategies

The teacher’s use of multiple approaches to mathematical problem solving is another vital strategy. Encouraging students to attempt different ways to solve problems and modelling a selection of approaches during instruction, could help teachers to develop a more interactive teaching style. This is likely to establish an open classroom thinking culture where students feel more comfortable to explore mathematics in ways that suit them better than traditional approaches.

We have thus considered three key extrinsic strategies for problem solving and the use of each strategy in having potential to address multiple risk factors for math anxiety. Extensive overlap exists within the dimensions of Table I. All of the selected strategies have potential to enhance the teacher’s mathematical content knowledge. In the next section we will similarly consider three key intrinsic strategies in Table II to explore the results of this review further.
Table II: Potential Intrinsic Math anxiety Risk factors and Strategies

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Students’ Emotions</th>
<th>Student’s Confidence/Self-Perception</th>
<th>Student’s Conceptual Understanding of Math</th>
<th>Student's Gender</th>
<th>Stereotypic Beliefs</th>
<th>Student’s Preferred Styles of Learning</th>
<th>Student’s Belief about Math</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple Representations</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Mathematical Resilience</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Estimation Skills</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Help-Seeking</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Cooperative Learning</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Informal Methods</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Mathematical resilience

Developing a persistent attitude to mathematical problem solving has been identified as an important strategy in managing math anxiety. If students learn to build resilience this could lead to students managing their emotions more effectively, harnessing their power to stimulate exploration of the problem domain and sets of possible solutions. The strength and range of emotions associated with mathematics has been previously emphasised. However, students who can moderate their emotional responses to mathematical problem solving situations are also more likely to develop increased self-confidence in mathematics.

Multiple representations

Using multiple ways to represent key elements of mathematical problems is a valuable strategy to help manage student math anxiety. Typically students construct diagrams or other visual representations of the problem to make the solution more transparent and aid improved conceptual understanding of mathematics. For example, graphing an algebraic function helps students visualise its characteristic behaviour over a range of values.

Cooperative learning

Cooperative learning, especially in small groups, was identified as another key strategy for students in managing math anxiety. Collaboration in classroom work, whether face-to-face or online is helpful in facilitating productive problem solving and helping students to feel less overwhelmed by mathematical tasks. This approach to mathematics could also assist students to access their preferred styles of learning by providing opportunities to explore different means of interaction, share ideas, and offer support to each other. By working with their peers
students become exposed to a wider variety of learning styles than they would if taught mathematics in a traditional way.

We have considered three key extrinsic strategies from Table II for problem solving and the use of each strategy in having potential to address multiple risk factors for math anxiety. Similarly to Table I, extensive overlap exists within the dimensions of Table II. All of the selected strategies have potential to enhance the students’ self-confidence and potentially improve their self-perception as learners of mathematics.

In summary, we have surveyed a selection of key intrinsic and extrinsic strategies and considered how each strategy could address multiple risk factors. Just as one strategy can be used to address multiple risk factors, one risk factor may also be addressed with multiple strategies.

**Implications for practice**

The pervasive nature of math anxiety for many students and teachers, across multiple academic levels, countries and cultures cannot be underestimated. The findings endorse the importance of the teacher in addressing their own and their students’ Math anxiety. Improving teacher’s mathematical content knowledge stands out clearly as a key issue in this investigation. Sound content knowledge leads to amelioration of affective dispositions towards math and more appropriate and positive teaching strategies. Practitioners who are teaching with modern approaches also have a responsibility to regularly acquaint themselves with emerging research in this field. The operation of power relations in mathematics education is also important for teachers to be aware of and handle sensitively. Although much of the literature focuses on the teacher as the main source of strategies, it is vital that students’ responsibilities and initiatives are not diminished.

Valuing common sense and multiple informal methods of developing solutions to mathematical problems feature strongly in this discussion, and are valuable resources for informed practitioners to draw on in their teaching. However, selecting appropriate everyday examples of mathematics to bridge the formal-informal knowledge divide is a challenging endeavour. Examples and analogies need to be sufficiently realistic and accurate to represent the essential features of the concept being explained. However, too much emphasis on physical representations of concepts can weaken student’s appreciation of the power of mathematical abstraction. There is evidence that such emphases have trade-offs which must be acknowledged (Koedinger, Alibali & Nathan, 2008; McNeil, Uttal, Jarvin & Sternberg, 2009).

**Conclusion**

The central challenge which guided this review of literature was to better understand the association between math anxiety risk factors and the strategies used to address them. This review confirmed the highly complex multidimensional nature of math anxiety and that a single strategy could address multiple risk factors. Mathematical content knowledge of teachers should be a significant component of sector-wide strategies to deal with math anxiety. Teacher professional development, increased awareness of the power of informal methods and multiple approaches to mathematical problem solving is needed to support efforts to reduce math anxiety. The proposed model of risk factors and strategies provides a basis for further quantitative research in order to establish measures of statistical validity, reliability and applicability in practical contexts.
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